The free RainOffT model, useful for analyzing the hydrology of subsurface drainage systems in transient (non-steady) state.

R.J.Oosterbaan

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Abstract

The free RainOff model software was originally designed to analyze the hydrology of watersheds (rainfall catchment areas). However it can also be used to simulate the discharge and depth of the groundwater table in agricultural subsurface drainage systems, This simulation can be done upon entering, amongst other, rainfall, irrigation and evaporation data as well as the parameters of the system. When a discharge record is available, while the drainage parameters are unknown, these data can be used to determine the reaction factor (reservoir response function) that can be employed for further analysis and simulation of the hydrological phenomena.

This article discusses the principles and algorithms of the RainOffT program and provides examples using data provided in literature.

Contents

1. Introduction

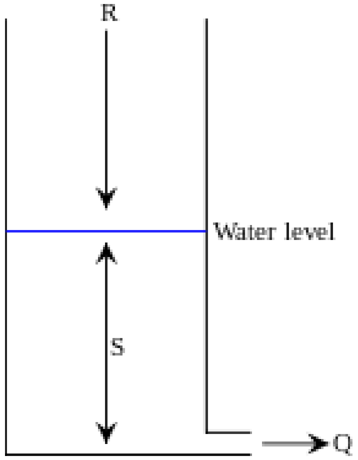
The free RainoffT program [Ref. 1] has initially been used to analyze the rainfall – runoff relations of a small valley in Sierra Leone [Ref. 2] and in Germany [Ref. 3]. Later, it was used to analyze agricultural subsurface drainage systems [Ref. 4].

In this article the hydrology of a subsurface drainage system in relation to the characteristics of a drainage system is dealt with using data provided by Ritzema [Ref. 5].

2. Principles of RainOff

2.1 Linear and non-linear reservoir

RainOff is built on the principles of a non-linear reservoir, an extension of the linear reservoir. The linear reservoir was described by D.A.Kraijenhof van de Leur [Ref. 6] and its principles are given in *figure 1*.

	R = recharge {dimension L/T} S = storage {dimension L} Q = discharge, runoff {dimension L/T}	***Figure. 1*.** *The concept of a linear reservoir.*

The reservoir response function is:

$Q = \alpha . S$ (Eq. 1)

where α = a constant reaction factor {1/T}
Differentiating S to time T gives

$dS/dT = d(Q/\alpha)/dT = R - Q$ (Eq. 2)

Integrating Eq. 2 with limits Q_1, Q_2, T_1 and T_2 yields:

$Q_2 = Q_1 \exp\{-\alpha (T_2 - T_1)\} + R\,[1 - \exp\{-\alpha (T_2 - T_1)\}\,]$ (Eq. 3)

where Q_2 and Q_1 are Q at time T_2 and T_1 respectively.

With Equation 3 the discharge Q_2 can be calculated from R, Q_1, α, and the time difference.

This concept is often to simple to characterize the watershed as its reaction factor is usually more complicated. Therefore Nash [Ref. 7] employed a cascade of linear reservoirs, one reservoir emptying into the next, while Kraijenhoff [Ref. 6] used a number of parallel reservoirs over which the rainfall is distributed in some proportion, while the reservoirs joined their discharge.

In hydrology, the concept of non-linear reservoirs has seldom been applied. Instead of a reservoir with a constant reaction factor, one could employ a non-linear reservoir with a reaction factor that changes with storage (*figure 2*) instead of being a constant, thus avoiding the difficulty of dealing with a series of reservoirs.

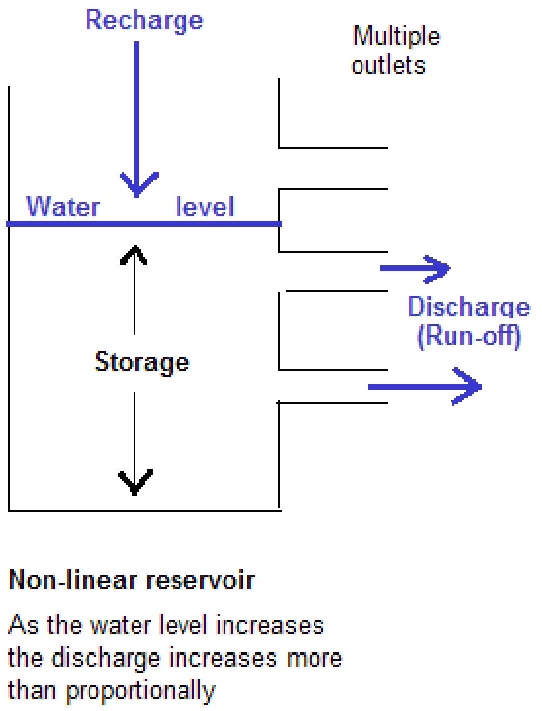


Figure 2.

A non-linear reservoir with multiple outlets whereby the discharge increases more than proportionally with the storage.

The equivalents of equation 1, 2 and 3 for the non-linear reservoir are equations 4, 5 and 6 as follows [Ref. 8]:

$$Q = (A.Q + C).S \qquad \text{(Eq. 4)}$$

$$dS/dt = R-(A.Q+C).S = R - A.Q.S + C.S \qquad \text{(Eq. 5)}$$

$$Q_2 = Q_1 \exp \{ -(A.Q_1+C).(T_2-T_1) \} + R[1-\exp\{-(A.Q_1+C).(T_2-T_1)\} \qquad \text{(Eq. 6)}$$

The reaction factor (or reservoir response function) can now be written as

$$\alpha = A.Q + C \qquad \text{(Eq. 7)}$$

It is no longer a constant, but it depends on the discharge. The factor B and the term C are found by RainOffT with a numerical (calibration) method, varying the B and C values and selecting the combination that maximizes the fit of the simulated discharge/runoff in time to the observed one.

The values B and C represent the properties (characteristics) of the precipitation catchment area (watershed), which needs only two parameters.

It is also possible to use a quadratic α function: $\alpha = A.Q^2 + B.Q + C$ [Ref. 1]. The software for this case is called RainOffQ. In some cases it gives a still better result.

2.2 Recharge

The recharge depends on the rainfall and the escape factors like evaporation and percolation to an aquifer with natural drainage. When the percolation is taken negative it will represent upward seepage from the aquifer. The rainfall enters a pre-reservoir with a storage function as shown in *figure 3*.

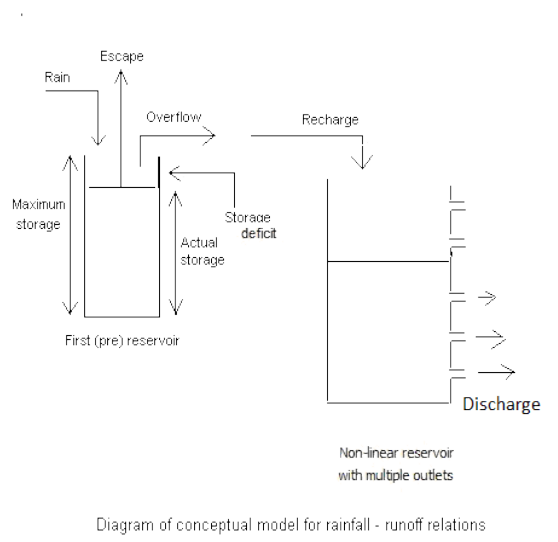


Figure 3.

The second (main) reservoir is preceded by a pre-reservoir that provides the net recharge after deducting evaporation (escape) and replenishment of the soil moisture

The "escape" usually consists of evaporation, but it may include percolation to the aquifer and natural drainage, while upward seepage from the aquifer can be considered as a negative Escape. The Recharge is thus found from:

Recharge = Overflow = Rain – Escape – Storage Deficit. (Eq. 8)

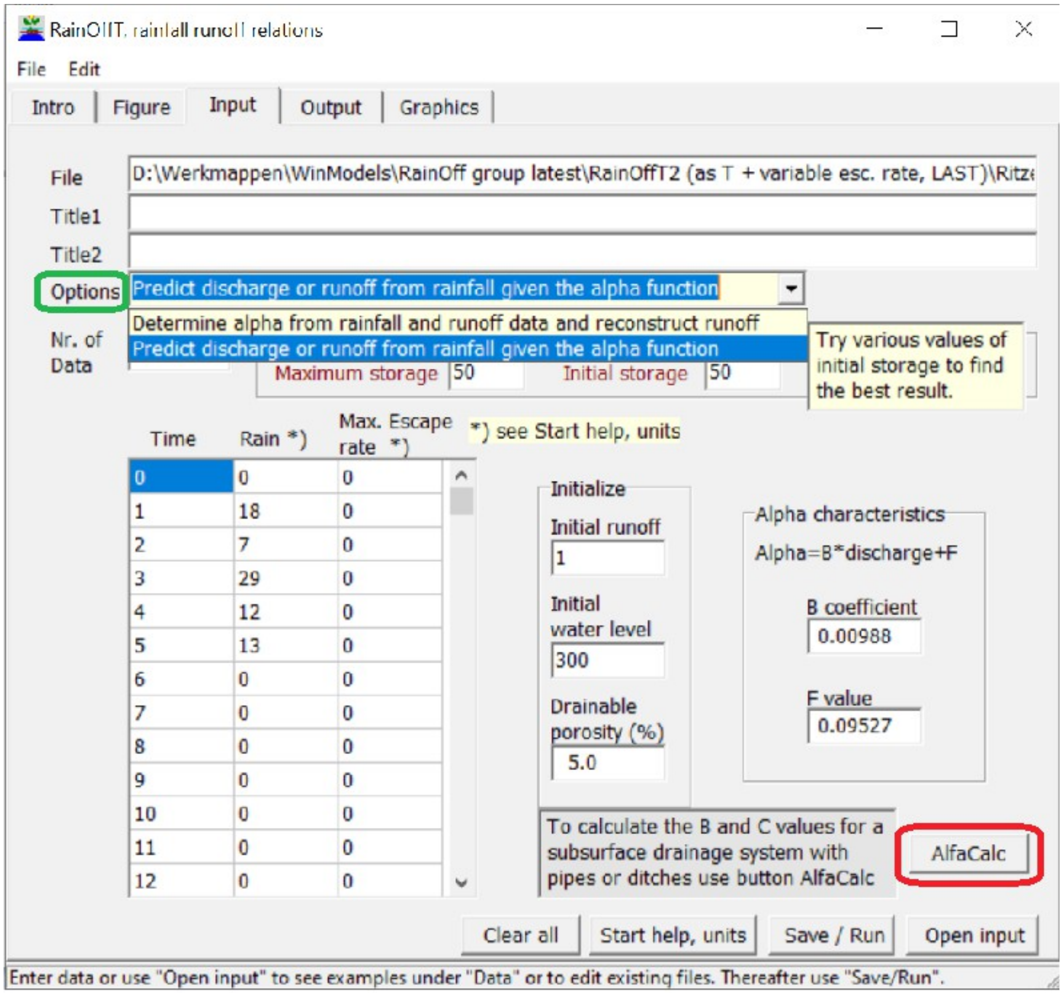
During rainy periods the Storage Deficit can become zero and the Recharge will equal the Rainfall less Evaporation. In dry periods the Escape may exceed the Rainfall and the Storage Deficit will then increase.

3. Examples based on data from literature

The data used in these example were taken from Ritzema [Ref. 5].

3.1 Parameter determination from drainage system properties

Figure 4 shows the input menu for the “Predict” option (green block). Further it exhibits an “AlfaCalc” button (red block) that gives the possibility to calculate the Alpha function having the factor B and the constant C.



File: D:\Werkmappen\WinModels\RainOff group latest\RainOffT2 (as T + variable esc. rate, LAST)\Ritz

Title1

Title2

Options: Predict discharge or runoff from rainfall given the alpha function

Determine alpha from rainfall and runoff data and reconstruct runoff

Predict discharge or runoff from rainfall given the alpha function

Nr. of Data

Maximum storage: 50

Initial storage: 50

Try various values of initial storage to find the best result.

*) see Start help, units

Time	Rain *)	Max. Escape rate *)
0	0	0
1	18	0
2	7	0
3	29	0
4	12	0
5	13	0
6	0	0
7	0	0
8	0	0
9	0	0
10	0	0
11	0	0
12	0	0

Initialize

Initial runoff: 1

Initial water level: 300

Drainable porosity (%): 5.0

Alpha characteristics

Alpha=B*discharge+F

B coefficient: 0.00988

F value: 0.09527

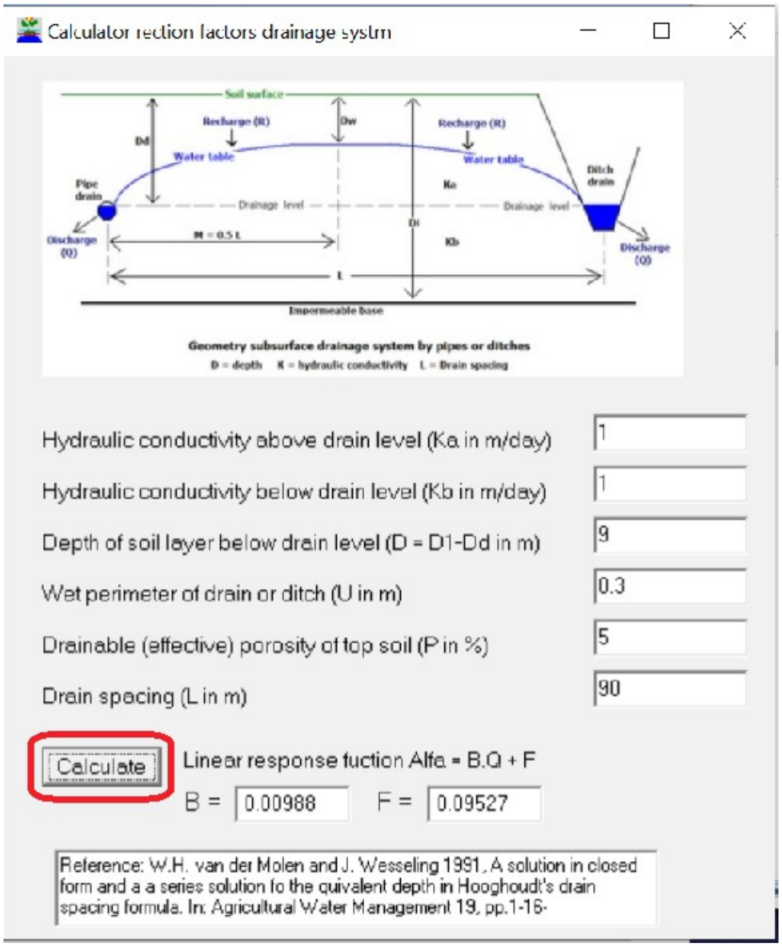
To calculate the B and C values for a subsurface drainage system with pipes or ditches use button AlfaCalc

Enter data or use "Open input" to see examples under "Data" or to edit existing files. Thereafter use "Save/Run".

Figure 4. *Input menu for the “Predict” option (green square) with an “AlfaCalc” button (red square) that gives the possibility to calculate the Alpha function (equation 7) having a coefficient B and constant value C. The data can be copied from a spreadsheet like Excel.*

Upon clicking the “AlphaCalc” button, a new screen is opened as depicted in *figure 5*.

The screen shows an illustration of subsurface drainage parameters, a table where these parameters can be filled in plus a “Calculate” button that produces the Alfa parameters (orange block).



Calculator rection factors drainage systm

Hydraulic conductivity above drain level (Ka in m/day): 1

Hydraulic conductivity below drain level (Kb in m/day): 1

Depth of soil layer below drain level (D = D1-Dd in m): 9

Wet perimeter of drain or ditch (U in m): 0.3

Drainable (effective) porosity of top soil (P in %): 5

Drain spacing (L in m): 90

Linear response fuction Alfa = B.Q + F

B = 0.00988 F = 0.09527

Reference: W.H. van der Molen and J. Wesseling 1991. A solution in closed form and a a series solution fo the quivalent depth in Hooghoudt's drain spacing formula. In: Agricultural Water Management 19, pp.1-16-

Figure 5. *This figure shows an illustration of subsurface drainage properties, a table where these properties can be filled in plus a “Calculate” button that produces the B and C values of the Alfa function. Here it reads: Alfa = 0.00988 Q + 0.0953*

The calculated Alfa parameters in *figure 5* are automatically transposed to the input screen, see figure 4. The computation of the Alfa parameters B and C is explained in the Appendix.

During the calibration phase (next section) we will see that the calibrated Alpha parameters are practically the same as those revealed *in figures 4 and 5*.

Figure 7 depicts the results of the prediction of the drain discharge based on drainage system properties, while *figure 8* compares the predicted and measured drain discharge.

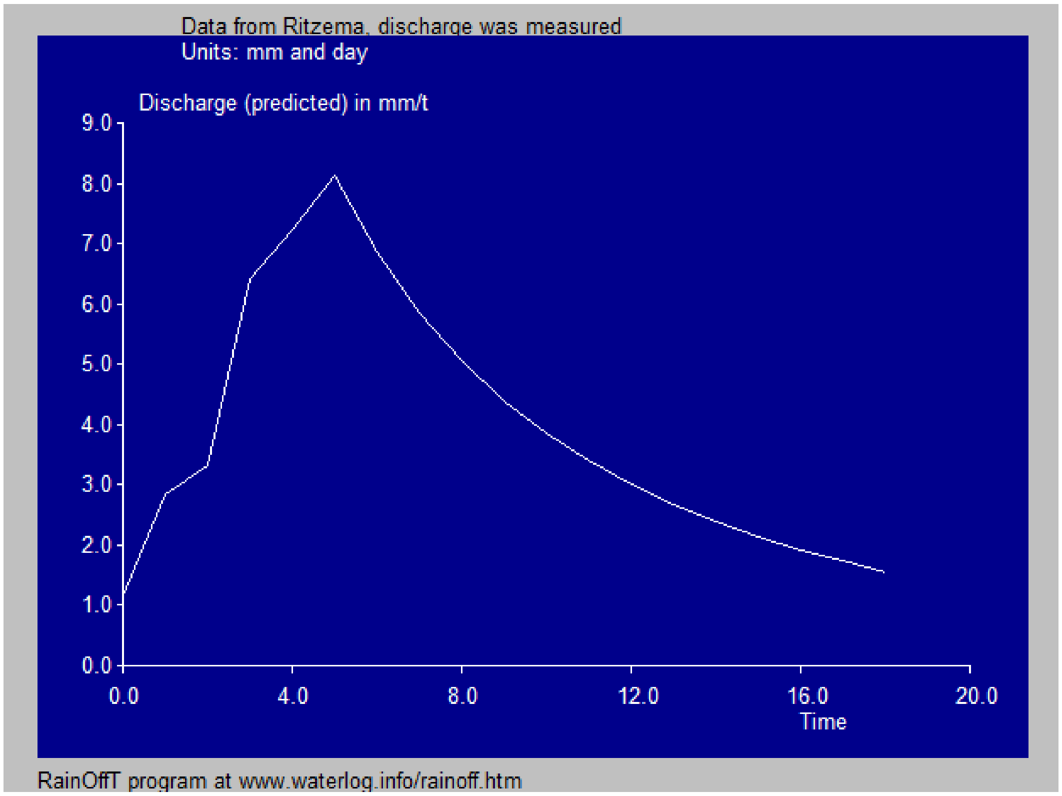


Figure *7. Predicted* ***discharge*** *on the basis of the properties of the subsurface drainage system (*figure 5*). Alfa = 0.00988 Q + 0.0953*

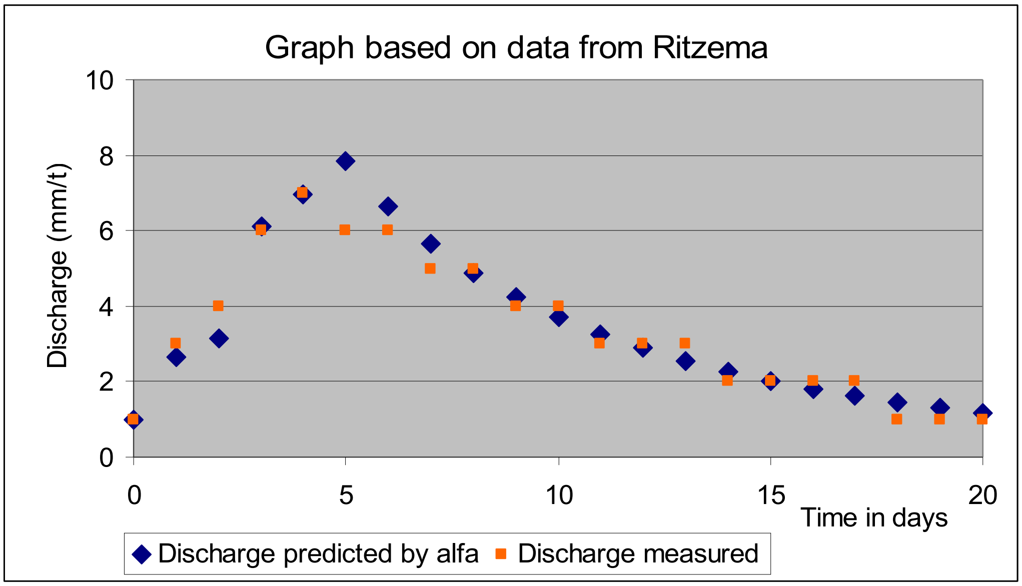
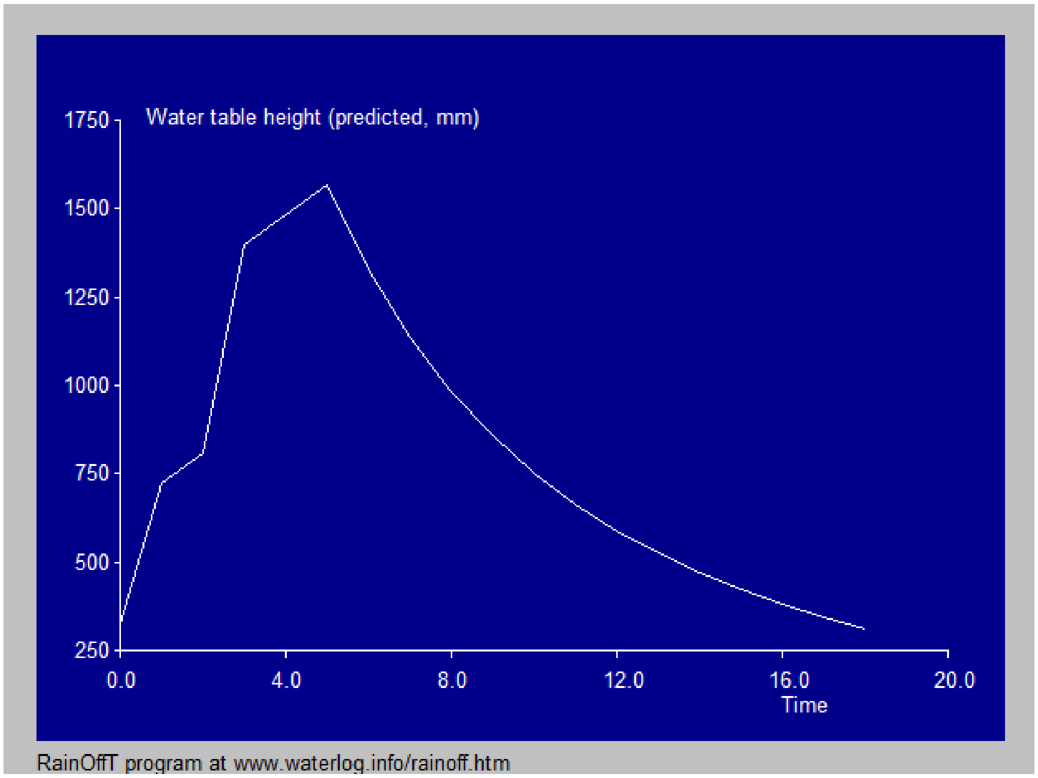


Figure 8. *Comparison of the predicted* ***discharge*** *on the basis of the properties of the subsurface drainage system (*figure 7*) with the measured discharge. The agreement is quite high.*

Figure 9 depicts the results of the prediction of the ground water table based on drainage system properties, while *figure 10* compares the predicted and measured water table.



RainOffT program at www.waterlog.info/rainoff.htm

Figure 9*. Predicted level of the* ***water table*** *on the basis of the properties of the subsurface drainage system (*figure 5*). Alfa = 0.00988 Q + 0.0953*

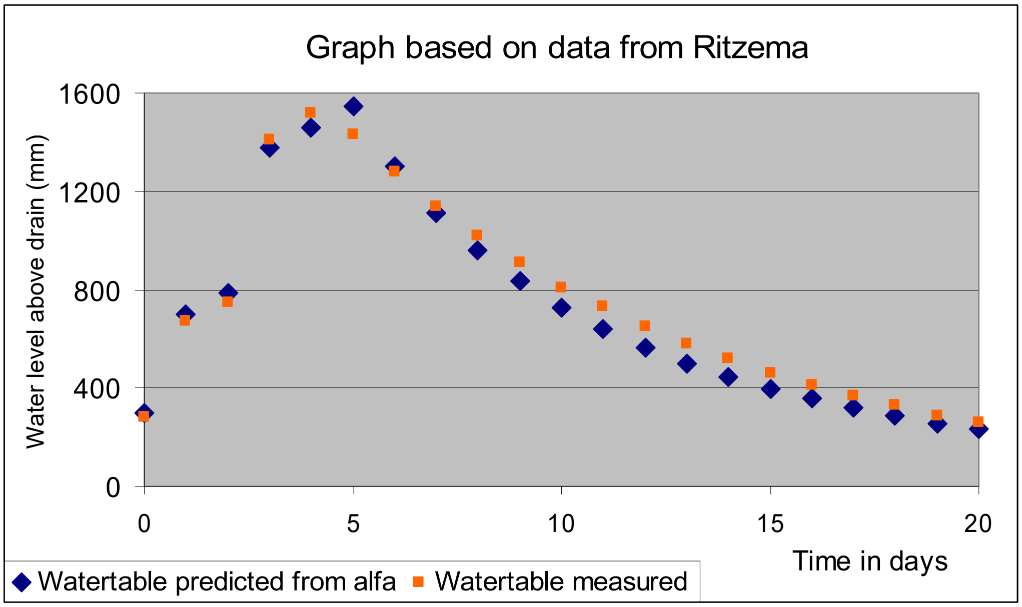
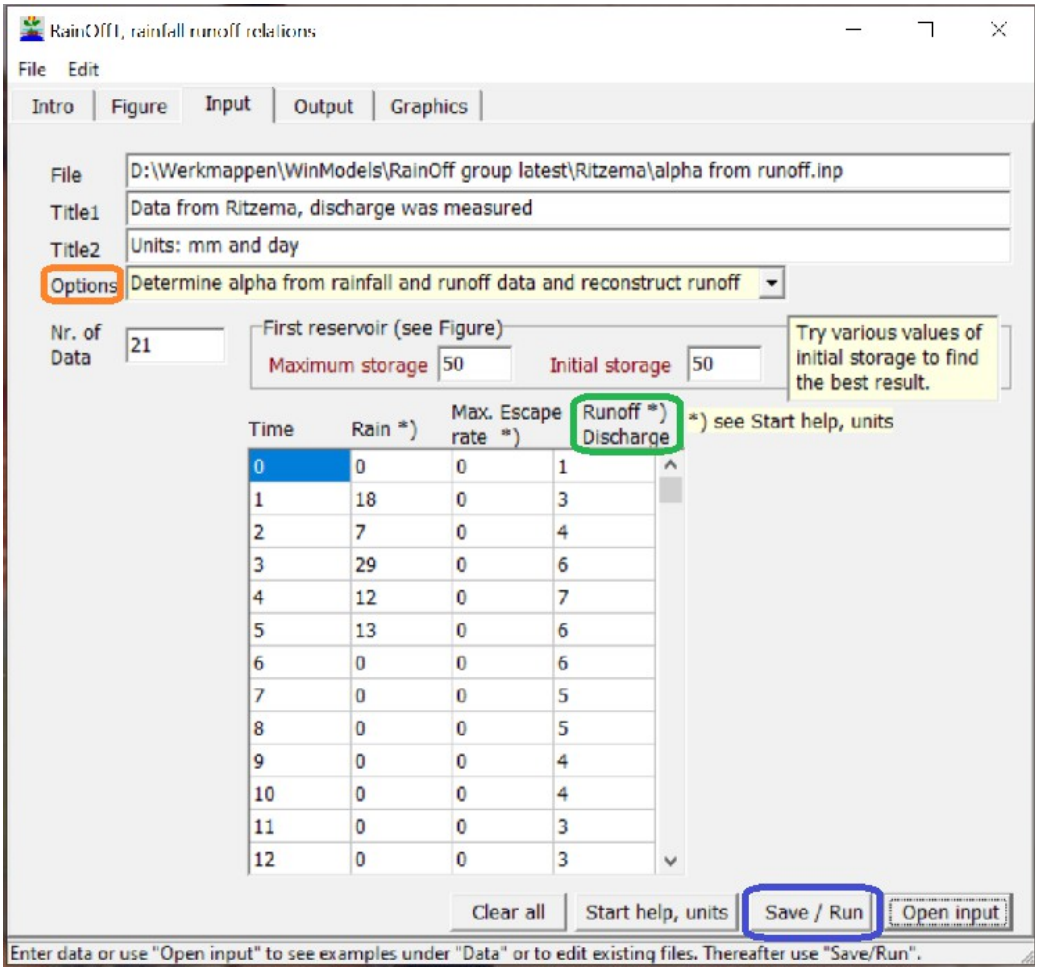


Figure 10. *Comparison of the predicted level of the* ***water table*** *on the basis of the properties of the subsurface drainage system (*figure 9*) with the measured level. The agreement is quite high.*

3.2 Parameter determination by discharge calibration

Instead of determining the alpha function from the parameters of the drainage system, it is also possible to derive the response function from the observed discharge data. The RainOffT program uses a range of B and C values and accepts the combination yielding the minimum value of the sum of the squares of the differences between the calculated and observed discharges.

The input menu for this procedure is depicted in *figure 11*. It shows that the option “Determine” is used instead of the option “Predict” in *figure 4*.



File: D:\Werkmappen\WinModels\RainOff group latest\Ritzema\alpha from runoff.inp

Title1: Data from Ritzema, discharge was measured

Title2: Units: mm and day

Options: Determine alpha from rainfall and runoff data and reconstruct runoff

Nr. of Data: 21

First reservoir (see Figure)

Maximum storage: 50

Initial storage: 50

Try various values of initial storage to find the best result.

*) see Start help, units

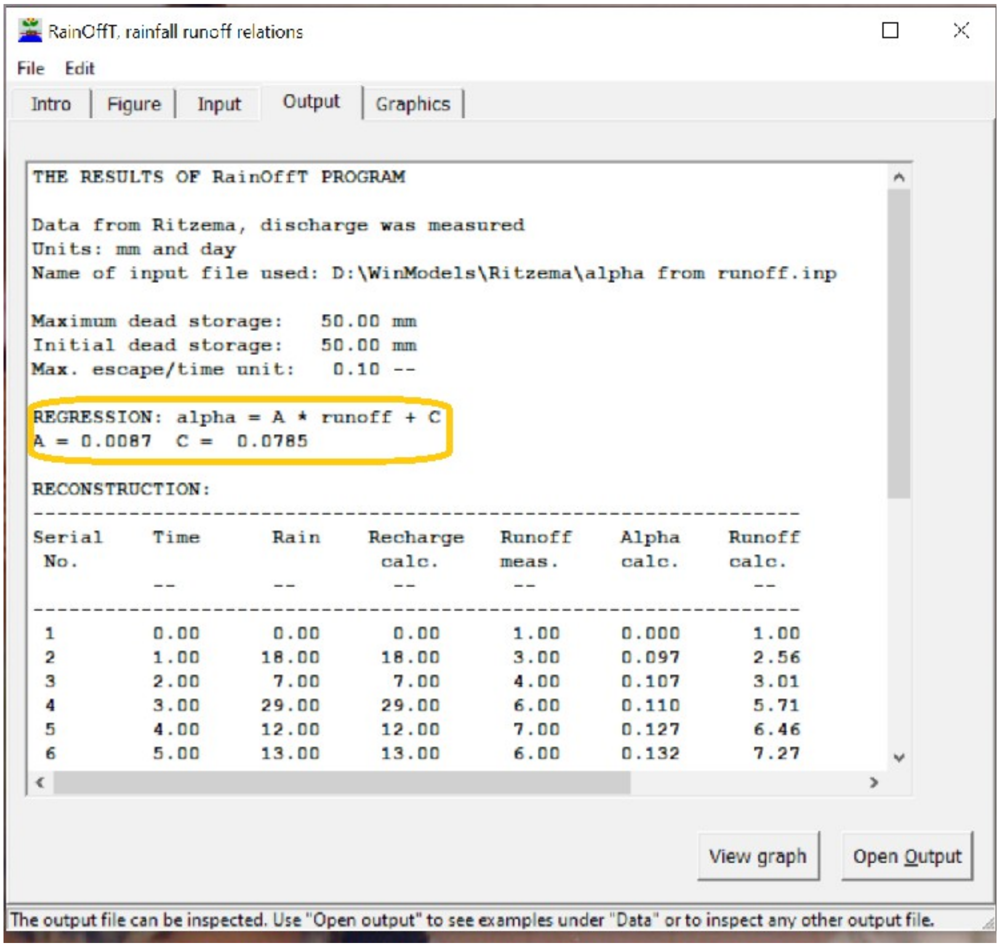
Time	Rain *)	Max. Escape rate *)	Runoff *) Discharge
0	0	0	1
1	18	0	3
2	7	0	4
3	29	0	6
4	12	0	7
5	13	0	6
6	0	0	6
7	0	0	5
8	0	0	5
9	0	0	4
10	0	0	4
11	0	0	3
12	0	0	3

Enter data or use "Open input" to see examples under "Data" or to edit existing files. Thereafter use "Save/Run".

Figure 11. *Screen print of the input menu for the option “Determine” (orange block). With this option the discharge data need to be given (green block). After completing the input data use the Save / Run button (blue block) to save the data and perform the calculations. The data can be copied from a spreadsheet like Excel*

A version (DrainCalc) with the additional option to determine alpha from rainfall and measured depths of the water table (hence not only from the measured runoff/discharge data) and reconstruct the depth is also available [*Ref. 9*].

The RainOffT output results are summarized in *Figure 12*.



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THE RESULTS OF RainOffT PROGRAM

Data from Ritzema, discharge was measured
Units: mm and day
Name of input file used: D:\WinModels\Ritzema\alpha from runoff.inp

Maximum dead storage:   50.00 mm
Initial dead storage:   50.00 mm
Max. escape/time unit:   0.10 --

REGRESSION: alpha = A * runoff + C
A = 0.0087  C =  0.0785

RECONSTRUCTION:
-----------------------------------------------------------------
Serial     Time       Rain     Recharge   Runoff     Alpha     Runoff
 No.                            calc.      meas.     calc.      calc.
           --         --         --         --                   --
-----------------------------------------------------------------
 1         0.00       0.00       0.00       1.00     0.000       1.00
 2         1.00      18.00      18.00       3.00     0.097       2.56
 3         2.00       7.00       7.00       4.00     0.107       3.01
 4         3.00      29.00      29.00       6.00     0.110       5.71
 5         4.00      12.00      12.00       7.00     0.127       6.46
 6         5.00      13.00      13.00       6.00     0.132       7.27
```

The output file can be inspected. Use "Open output" to see examples under "Data" or to inspect any other output file.

Figure 12. *RainOffT output for the case shown in* figure 11. *The alpha function has been optimized to Alpha = 0.0087 Q + 0.0785. This is somewhat different from the alpha function found on the basis of the characteristics of the drainage system: Alfa = 0.00988 Q + 0.0953*

The difference between the two alpha functions mentioned in the subscript of *figure 12* is possibly due to an irregularity in the observed discharge-recharge data as demonstrated in *figure 13*.

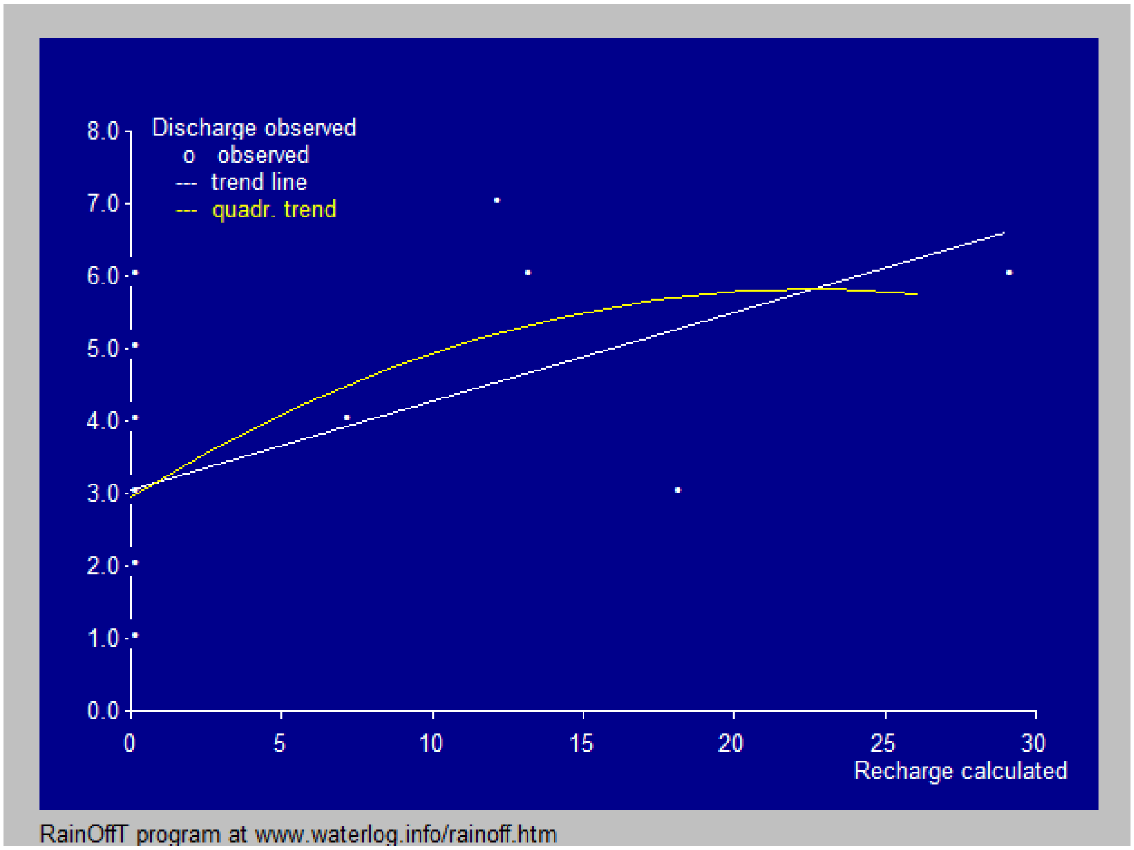
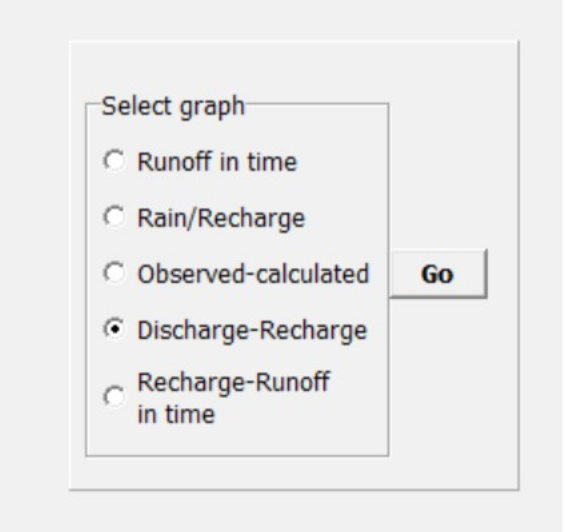


Figure 13. *The relation between observed discharge and recharge has been obtained with a linear regression (white line) and a quadratic regression (yellow curve). The yellow curve shows a descending trend of the discharge at the higher recharge values, which is not logical. Especially the point (Y, X) = (2.9, 18) lies exceptionally low, which causes the determination of the alfa function to be less secure.*

The graph of the type seen in *figure 13* can be produced using the discharge-recharge selection in the table of graphics choices of RainOffT as specified in *figure 14*.



Select graph

○ Runoff in time

○ Rain/Recharge

○ Observed-calculated

● Discharge-Recharge

○ Recharge-Runoff in time

Figure 14.

Graphic selection choices offered by RainOffT.

For *figure 13* the Discharge-Recharge choice has been selected

To check the influence of the difference alfa values, the RainOffT program was executed with the "Predict" option as shown in *figure 4* using an alfa function found with the optimization procedure given in *figure 12* as Alfa = 0.0087 Q + 0.0785, (Alfa2). and in *figure 15* the results are compared with the previous outcomes in *figure 8* for the observed discharge and the one calculated from the parameters of the drainage system: Alfa = 0.00988 Q + 0.0953 (Alfa1).

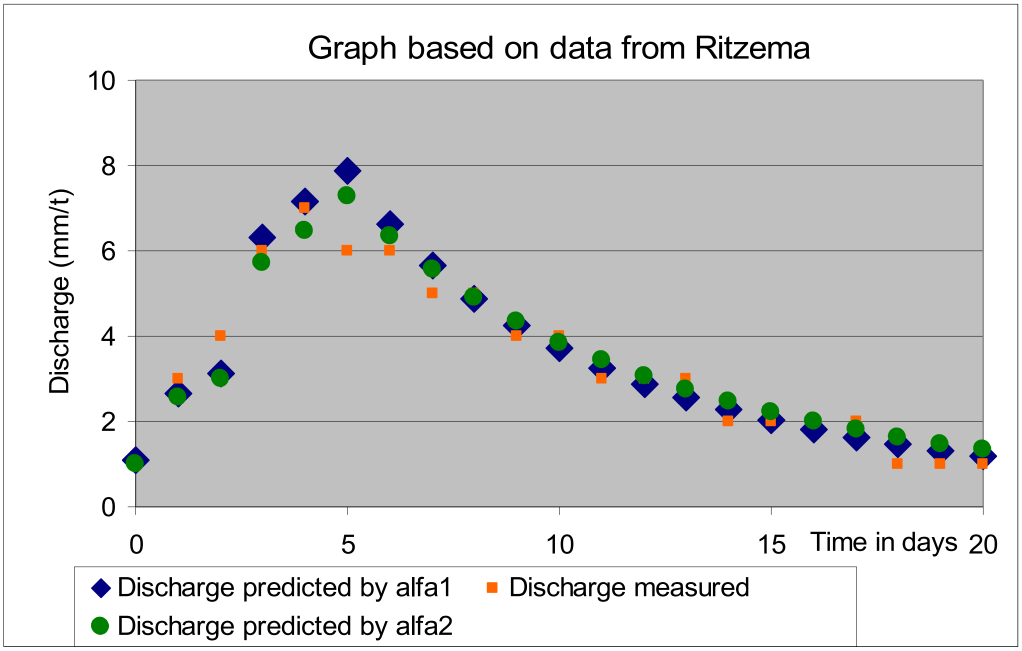


Figure 15. *The simulated discharges depicted in* figure 8 *by optimization (here named alfa1), and the observed ones are combined with the discharges obtained by prediction (here named Alfa2). The deviations are relatively small, hence the difference between the two Alfa functions is acceptable and does not lead to large errors. Also, the influence of the irregularity demonstrated in* figure 13 *appears to be not very influential. One can also conclude that the data on the parameters of the drainage system are dependable as they lead to reliable discharges.*

6. Conclusion

The RainoffT model is conceptual in that it uses the non-linear reservoir principles which makes it feasible to apply the reservoir responsefunction (reaction function) to practical situations like the influence of agricultural subsurface drainage systems on the fluctuating level of the groundwater table and to determine the transient drain discharge depending on the recharge by irrigation and/or rainfall less the evaporation and the water shortage during dry spells.

The reservoir response function can be determined by calibrating the simulated to the observed drain discharge or by interpreting the physical properties (characteristics, parameters) of the drainage system.

From the examples given it can be concluded that the correspondence between measured and simulated discharge, as well as the water level, is high.

RainoffT is able to detect inconsistencies in the data set and to check their influence on the outcomes, which influence appeared to be small in the given examples. To the contrary, in the case the German Hornseelbach watershed [Ref. 3], the irregularities detected had a considerably negative influence.

5. References

[Ref. 1] RainOff, RainOffT and RainOffQ, model software for rainfall-runoff analysis:
https://www.waterlog.info/rainoff.htm

[Ref. 2] R.J. Oosterbaan, 2019. *Rainfall-runoff relations of a small valley in Sierra Leone with a non-linear reservoir model*. International Journal of Environmental Science, ***4**, 1-9 , 2019.*
On line: https://www.waterlog.info/pdf/SLeone.pdf
or*:*
https://www.researchgate.net/publication/332466264_RAINFALL-RUNOFF_RELATIONS_OF_A_SMALL_VALLEY_ASSESSED_WITH_A_NON-LINEAR_RESERVOIR_MODEL

[Ref. 3] Rainfall and runoff data of the "Herbornseelbach" catchment (watershed), Hesse, Germany, evaluated with the RainOff model by calibration and validation of catchment parameters. On line: https://www.waterlog.info/pdf/Hesse Herborn.pdf
or:
https://www.researchgate.net/publication/349662750_Rainfall_and_runoff_data_of_the_Herbornseelbach_catchment_watershed_Hesse_Germany_evaluated_with_the_RainOff_model_by_calibration_and_validation_of_catchment_parameters

[Ref. 4] Simulating subsurface drain discharge and depth of the water table in transient (non-steady) state using the RainOff model. On line:
https://www.researchgate.net/publication/343943124_Simulating_subsurface_drain_discharge_and_depth_of_the_water_table_in_transient_non-steady_state_using_the_RainOff_model

[Ref. 5] H.P. Ritzema, 1994. *Subsurface Flow to drains*. Chapter 8 in: H.P. Ritzema (editor), Drainage Principles and Applications, ILRI Publication 16. International Institute for Land Reclamation and Improvement, Wageningen, The Netherlands. On line:
http://www2.alterra.wur.nl/Internet/webdocs/ilri-publicaties/publicaties/Pub162/pub162-h1.0.pdf
or:
https://www.researchgate.net/publication/272483377_Subsurface_flow_to_drains

[Ref. 6] J.E.Nash, 1958. *Determining runoff from rainfall*. Proc. Inst. Civ. Engs. 10 : p. 163 – 184. On line: https://www.icevirtuallibrary.com/doi/10.1680/iicep.1958.2025
Software: https://www.waterlog.info/nashmod.htm

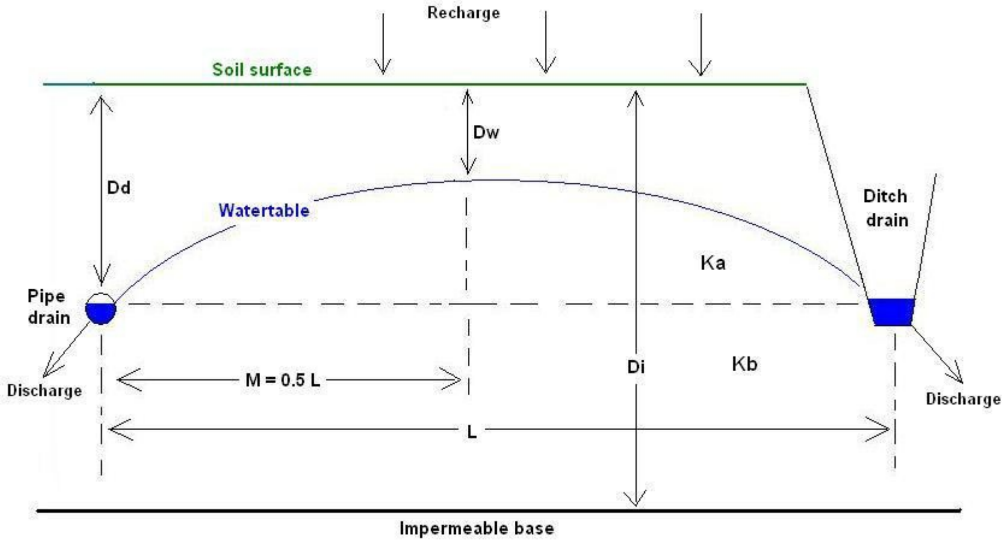
[Ref. 7] D.A.Kraijenhoff van de Leur, 1958. *A study of non-steady state groundwater flow with special reference to a reservoir coefficient*. De Ingenieur 70: p. 387 – 394. On line:
https://library.wur.nl/WebQuery/hydrotheek/604860

[Ref. 8] Rainfall-runoff model with non-linear reservoir. On line:
https://www.waterlog.info/pdf/reservoir.pdf

[Ref. 9] DrainCalc, free software for the runoff in watersheds or the discharge in subsurface drainage systems. Extension to the RainOff model (see Ref. 4) incorporating the analysis of the depth of the water table in addition to the runoff/discharge analysis. Download from
https://www.waterlog.info/draincalc.htm

6. Appendix (computation of Alfa parameters for a drainage system)

The parameters (characteristics) of an agricultural subsurface drainage system are illustrated in figure A.



Geometry subsurface drainage system by pipes or ditches
D = depth   K = hydraulic conductivity   L = drain spacing

Figure A. Parameters of **of an agricultural subsurface drainage system**

In the situation of figure A, the steady state drainage equation of Hooghoudt is applicable [Ref. 5].:

$$Q = \frac{8Kb.De.H}{L^2} + \frac{4Ka.H^2}{L^2}$$

The height (H in m) of the water table midway between the drains above drain level equals Dd-Dw in figure A.
Ka and Kb = hydraulic conductivity above and below drain level respectively (m/day)
L = drain spacing (m)
De = equivalent depth of the impermeable layer below drain level. It depends on the actual depth Da = Di – Dd (see figure A) of the impermeable layer below drain level. The mathematical expression of De in terms of Da is shown on the next page.
Q is expressed in m/day.

The drainable storage S of water midway between the drains equals S = Pd.H where Pd is the drainable porosity (in m/m) of the soil, also called effective porosity. In clay soils it normally varies between 2 and 4%, in loamy soils it may vary from 3 to 5% and in sandy loams it may range from 4 to 6% and in sandy soil from 5 to 10%

Writing $Q = \alpha.H$ we find

$$\alpha = \frac{8Kb.De}{L^2} + \frac{4Ka.H}{L^2}$$

or:

$$\alpha = B + A.H$$

where:

$$B = 8Kb.De / L^2$$

$$A = 4Ka / L^2$$

yielding a reaction (response factor α) depending on the storage S (and therefore also on Q), so that we have a non linear reservoir.

In transient (un-steady state) the expressions of B and A need to be changed into [Ref. 5]:

$$B = \pi^2.Kb.De / Pd.L^2$$

$$A = 0,5\ \pi^2.Ka / Pd.L^2$$

Equivalent depth De

Reference: W.H. van der Molen and J. Wesseling 1991. A solution in closed form and a series solution for the thickness of the equivalent layer in Hooghoudt's drain spacing formula. Agricultural Water Management 19, pp. 1-16

$$De = \frac{\pi\ L/8}{\ln (L/U) + F(x)}$$

where U = wet circumference of the drain (m) and F(x) is a function of

$$x = 2\ \pi\ Da / L$$

When x>1 then:

$$F(x) = \frac{4e^{-2x}}{(1- e^{-2x})} + \frac{4e^{-6x}}{3(1- e^{-6x})} + \frac{4e^{-10x}}{5(1- e^{-10x})} + \ldots\ldots$$

For x<=1:

$$F(x) = \pi^2 / 4x + \ln (x/2\pi)$$

Note.

For a half full pipe drain $U = \pi r$ with r = drain radius. For a ditch drain U equals bottom width + twice the length of the part of the sides that is under water.